

High Velocity Oblique Cloud Collision and Star and Star Cluster Formation through Gravitational Instability of the Shock-Compressed Slab with Rotation and Velocity Shear

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ABSTRACT

We study the gravitational instability of an isothermal gaseous slab formed by cloud-cloud collision and compression at the cloud interface. The compressed gaseous slab rotates and has velocity shear except when the collision is not exactly head-on. The effects of the rotation and velocity shear on the gravitational instability have been evaluated for the first time. We obtained the growth rate of perturbations as a function of the wavelength for various gaseous slab models. We also obtained the changes in the density and velocity due to the perturbations. Two types of unstable modes are found; one is due to the self-gravity of the gaseous slab and the other is due to the velocity shear. The former instability leads to the formation of star and star clusters. Velocity shear decreases the growth rate of the former instability while it increases that of the latter instability. Rotation also decreases the growth of the former instability. As the slab grows in mass, the growth rate of the self-gravitational instability increases and eventually becomes greater than the growing timescale of the slab. Then a gravitationally bound condensation is formed and its mass is larger when the slab has stronger velocity shear and rotation.

Key words: Galaxies:star clusters—Hydrodynamics—Instabilities—ISM:clouds—Stars:formation

1. Introduction

The high-velocity cloud-cloud collision has been thought to be one of the important triggering mechanisms of the star and star cluster formation. When two clouds collide with supersonic velocities, dense gaseous slab bounded by shock waves forms at the interface of the colliding clouds. The dense gaseous slab grows in mass and becomes unstable against fragmentation because of the self-gravity. The fragments collapse and evolve into stars and star clusters.

Many evidences have been found for star formation triggered by the cloud-cloud collision. NGC1333 (Loren 1976), G110-13 (Odenwald et al. 1992), W49N (Serabyn et al. 1993), and Sagittarius B2 (Hasegawa et al. 1993) are nominated as the present sites of massive star formation triggered by cloud collision. The number density of Galactic H II regions is proportional to the square of the local gas density, ρ_{H_2} . This static suggests that substantial fraction of OB stars exciting the H II regions are formed by cloud-cloud collision (Scoville et al. 1986).

It is also proposed that the populous and globular star clusters are formed by high-velocity gas clouds collisions. Fujimoto and Noguchi (1990), Fujimoto and Kumai (1991a,b), Ashman and Zepf (1992), and Kumai, Basu and Fujimoto (1993) assessed the gas-dynamical circumstances of the Magellanic Clouds, M31, M33, M82, and other galaxies having young populous and globular star clusters, and found that these galaxies have large amount of gas in large-scale unorganized motion induced internally or externally. In a great contrast to these galaxies, our Galaxy has neither young globular clusters, nor large-scale unorganized motion of gas. From this comparison, they developed the model that unorganized motion with velocity greater than $\sim 50\text{km s}^{-1}$ creates strongly-compressed regions of gas through cloud-cloud collision, and trigger the star cluster formation (see, Gunn 1980, Sabano and Tosa 1985, and Kang et al. 1990

for another model of star cluster formation by cloud-cloud collision). In this model, the gas in the compressed slab fragments into clumps whose mass is comparable to that of the whole star cluster, and they cascade to smaller and smaller fragments, finally they form a gravitationally bound stellar systems, - globular clusters.

In the star and star cluster formation model by cloud collision one must take account of oblique collisions since they are much more frequent than head-on collisions. When two clouds collide obliquely, the shock compressed gas slab rotates and is stressed by the velocity difference in the tangential component (velocity shear). The rotation and velocity shear may affect the fragmentation of the shock compressed gas slab. The present paper evaluates the effects on the gravitational instability of the slab for the first time. It is shown that velocity shear increases the effective sound speed of the gas and accordingly the minimum mass for instability (the Jeans mass). Rotation also increases the minimum mass for the instability.

In section 2 we describe our slab-model which mimics the shock compressed gas formed by oblique collision of clouds. Our slab takes account of rotation of the slab and velocity shear in the slab. In section 3, we formulate a linear analysis of perturbation around the equilibrium state. The numerical results are shown in section 4 consisting of subsections 4.1 through 4.4 each of which are for non-rotating slab without shear, non-rotating slab with shear, rotating slab without shear, and rotating slab with shear, respectively. In section 5 we estimate the Jeans mass to apply the dispersion relation obtained in the previous section to star and star cluster formation. Section 6 is devoted to discussion and conclusion.

2. Dynamical Models for a Shock-Compressed Slab of Self-Gravitating Gas

In the supersonic cloud-cloud collision, a shock-compressed gaseous slab is formed at the interface of the two colliding clouds. (e.g., Stone 1970; Struck-Marcell 1982; Sabano and Tosa 1985). Figure 1 is the snapshot of a model of the oblique collision, with the Cartesian coordinates referred to the gaseous slab, where $x - y$ plane is taken on the interface and the z -axis is taken normal to it. The gaseous slab rotates around the y -axis with angular velocity Ω , and the compressed gas flows parallel to the x axis with velocities u_0 directed toward $x = +\infty$ in the space $z > 0$ and $x = -\infty$ in $z < 0$. The shear flow is assumed to be uniform in the x - and y - directions for simplicity, $u_0 = u_0(z)$. As shown in (c) of figure 1, the flow parallel to the x -axis is dominant over that to the z -axis, because the latter component is much reduced by the strong compression. We neglect the latter component in the present paper.

In order to simplify the problem, we take the following three approximations. First we neglect the high-temperature region immediately behind the shock wave and assume the whole shock-compressed slab to be isothermal. The high temperature region is small in mass because of fast radiative cooling therefrom. Second the gaseous slab is assumed to be uniform in the x - and y -directions. As noticed by Stone (1970), the loss of gas across the lateral edges is negligibly small during the period of the collision. Third the gaseous slab is assumed to be in equilibrium, although its column mass actually increases with time. This equilibrium state can be achieved, when the sound wave propagates from the shock front to the interface before the gaseous slab grows considerably. As Welter (1982) showed, it is actually realized in the supersonic collision of gas clouds. When half of the slab thickness and isothermal sound speed are denoted with L and c respectively, we have for the sound crossing

Fig. 1.— The obliquely colliding clouds. a: Snapshot of the ongoing collision. Thick arrows are a reference frame. The clouds move along horizontal lines and collide. We find a shock compressed slab at the interface of two clouds. b: A model of the shock-compressed slab (dark patch). Circles spread by a light patch denote the projected clouds on the $x - z$ plane. c: Gas flows in and out of the slab. The pre-shocked gas streams along the horizontal lines, and compressed by the oblique shock. Although the post-shocked gas flows along dashed arrows, we approximate the stream line by solid arrows parallel to the slab.

time t_s between $0 \leq z \leq L$,

$$t_s \simeq \frac{L}{c}. \quad (2.1)$$

Similarly the increasing time t_m of the slab mass is written as

$$t_m \simeq \sigma / \left(\frac{d\sigma}{dt} \right) \simeq \frac{\rho_1 L}{\rho_0 c M_\perp} \simeq M_\perp t_s, \quad (2.2)$$

where σ , ρ_0 , ρ_1 , and M_\perp denote respectively the column mass of the slab, preshock density, postshock density, and the Mach number of the gas incident on the shock front normally. Here we have used relations, $\sigma \sim 2\rho_1 L$, $d\sigma/dt \sim 2\rho_0 c M_\perp$, and $\rho_1/\rho_0 = M_\perp^2$. We have thus quasi-equilibrium when $t_s < t_m$, and $M_\perp > 1$, both of which hold in the present supersonic collision of gas clouds.

3. Normal Mode Analysis for a Plane-Parallel Compressed Gaseous Slab in Rotation and Nonuniform Parallel Streaming

We analyze linear stability of a self-gravitating gaseous slab with the shear along the x -axis and the rigid rotation around the y -axis (figure 1). The basic equations governing the motion of gas are written as,

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} + 2\mathbf{\Omega} \times \mathbf{u} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}) = -\nabla\psi - \frac{\nabla p}{\rho}, \quad (3.1)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \quad (3.2)$$

and

$$\triangle \psi = 4\pi G \rho, \quad (3.3)$$

where \mathbf{u} , $\mathbf{\Omega}$, \mathbf{r} , ψ , p , ρ are usual symbols, representing the velocity of gas, angular velocity of the slab, position vector, gravitational potential, pressure, and gas density, respectively. As noted in section 2 and shown in figure 1, the angular

velocity of the slab is parallel to the y -axis, and we have explicitly,

$$\mathbf{\Omega} = (0, \Omega, 0). \quad (3.4)$$

Since we assume an isothermal gas, the pressure is written as

$$p = c^2 \rho. \quad (3.5)$$

The dynamical quantities \mathbf{u} , ρ , ψ , p are expressed as the sum of those in an equilibrium state and the deviations from it, the former and latter of which are specified with suffix 0 and with hat and suffix 1, respectively, like for example,

$$\rho(\mathbf{r}, t) = \rho_0(z) + \hat{\rho}_1(\mathbf{r}, t). \quad (3.6)$$

The velocity in equilibrium, \mathbf{u}_0 , has only the x -component and is a function of z ,

$$\mathbf{u}_0 = [u_0(z), 0, 0]. \quad (3.7)$$

Note that the other equilibrium quantities p_0 and ψ_0 are also functions of z and independent of t , x and y , since we have assumed that the slab is homogeneous in the x - and y -directions.

3.1. Equilibrium Configuration of the Rotating Gaseous Slab with Shear

Substituting equation (3.6) and similar equations for other variables, into equations (3.1), and (3.3), and using equations (3.4), (3.5), and (3.7), we derive the equations for the equilibrium state, by omitting the terms with hatted symbols,

$$2\Omega u_0 = \rho_0 \frac{d\psi_0}{dz} + c^2 \frac{d\rho_0}{dz}, \quad (3.8)$$

and

$$\frac{d^2 \psi_0}{dz^2} = 4\pi G \rho_0. \quad (3.9)$$

Equation of mass conservation (3.2) is automatically satisfied. Equation (3.8) represents that the Coriolis force per unit mass balances the gravitational acceleration and pressure gradient. Three variables, u_0 , ρ_0 , and ψ_0 are constrained by only two equations (3.8) and (3.9), and hence we have

infinitely large number of solutions. In this paper we restrict ourselves to the case that the density distribution has the same functional form to that given by Spitzer (1942),

$$\rho_0(z) = \rho_c \text{sech}^2\left(\frac{z}{H_R}\right). \quad (3.10)$$

Then we obtain,

$$u_0(z) = U \frac{H_R}{H} \tanh\left(\frac{z}{H_R}\right), \quad (3.11)$$

$$\psi_0(z) = 4\pi G \rho_c H_R^2 \ln\left[\cosh\left(\frac{z}{H_R}\right)\right], \quad (3.12)$$

with

$$H_R \equiv \frac{H}{\sqrt{1 - \Omega U H / c^2}} \quad (3.13)$$

with $\Omega U H / c^2 < 1$, and

$$H \equiv \frac{c}{\sqrt{2\pi G \rho_c}}, \quad (3.14)$$

where ρ_c is the density at $z = 0$, and H_R is a density scale height which tend to H as Ω to zero. The velocity shear is $du_0/dz = U/H$ on the $z = 0$ plane. The column density σ is given by

$$\sigma \equiv \int_{-L}^L \rho dz = 2\rho_c H_R \tanh\left(\frac{L}{H_R}\right). \quad (3.15)$$

3.2. Linear Perturbed Equations

Linearized equations of equations (3.1) to (3.3) and (3.5) are written as, by using equations (3.4), (3.6) and (3.7),

$$\begin{aligned} \frac{\partial \hat{u}_{x1}}{\partial t} + u_0 \frac{\partial \hat{u}_{x1}}{\partial x} + \hat{u}_{z1} \frac{\partial u_0}{\partial z} + 2\Omega \hat{u}_{z1} = \\ -\frac{\partial \hat{\psi}_1}{\partial x} - \frac{\partial}{\partial x} \left(\frac{\hat{p}_1}{\rho_0} \right), \end{aligned} \quad (3.16)$$

$$\frac{\partial \hat{u}_{y1}}{\partial t} + u_0 \frac{\partial \hat{u}_{y1}}{\partial x} = -\frac{\partial \hat{\psi}_1}{\partial y} - \frac{\partial}{\partial y} \left(\frac{\hat{p}_1}{\rho_0} \right), \quad (3.17)$$

$$\frac{\partial \hat{u}_{z1}}{\partial t} + u_0 \frac{\partial \hat{u}_{z1}}{\partial x} - 2\Omega \hat{u}_{x1} =$$

$$-\frac{\partial \hat{\psi}_1}{\partial z} - \frac{\partial}{\partial z} \left(\frac{\hat{p}_1}{\rho_0} \right), \quad (3.18)$$

$$\begin{aligned} \frac{\partial \hat{\rho}_1}{\partial t} + u_0 \frac{\partial \hat{\rho}_1}{\partial x} + \rho_0 \frac{\partial \hat{u}_{x1}}{\partial x} + \rho_0 \frac{\partial \hat{u}_{y1}}{\partial y} + \\ + \frac{\partial}{\partial z} (\rho_0 \hat{u}_{z1}) = 0, \end{aligned} \quad (3.19)$$

$$\Delta \hat{\psi}_1 = 4\pi G \hat{\rho}_1, \quad (3.20)$$

$$\hat{p}_1 = c^2 \hat{\rho}_1. \quad (3.21)$$

Since the equilibrium quantities are functions only of z , the perturbed ones can be expressed in the form of, for example,

$$\hat{\rho}_1(\mathbf{r}, t) = \rho_1(z) \exp[i(k_x x + k_y y - \omega t)]. \quad (3.22)$$

We introduce new variable $\mathbf{y}(z)$, consisting of four component, y_1 through y_4

$$\mathbf{y}(z) = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \equiv \begin{bmatrix} p_1 \\ i\rho_0 u_{z1}/\omega \\ \rho_c \psi_1 \\ \rho_c g_{z1} \end{bmatrix}, \quad (3.23)$$

where g_{z1} denotes the perturbed gravity in the z -direction,

$$g_{z1} \equiv \frac{d\psi_1}{dz}. \quad (3.24)$$

After straightforward manipulation we have for equations (3.16) through (3.21),

$$\frac{d\mathbf{y}}{dz} = \mathbf{A}\mathbf{y}, \quad (3.25)$$

where the 4×4 matrix \mathbf{A} has following elements

$$A_{11} = \frac{1}{\rho_0} \frac{d\rho_0}{dz} - \frac{2\Omega k_x}{k_x u_0 - \omega}, \quad (3.26)$$

$$\begin{aligned} A_{12} = -\omega(k_x u_0 - \omega) + \\ + \frac{2\Omega \omega}{k_x u_0 - \omega} \left(\frac{du_0}{dz} + 2\Omega \right), \end{aligned} \quad (3.27)$$

$$A_{13} = -\frac{2\Omega k_x}{k_x u_0 - \omega} \frac{\rho_0}{\rho_c}, \quad (3.28)$$

$$A_{14} = -\frac{\rho_0}{\rho_c}, \quad (3.29)$$

$$A_{21} = \left(\frac{c^2}{\omega}\right)(k_x u_0 - \omega) - \frac{1}{k_x u_0 - \omega} \left(\frac{k_x^2 + k_y^2}{\omega}\right), \quad (3.30)$$

$$A_{22} = \frac{1}{k_x u_0 - \omega} \left[\frac{d}{dz}(k_x u_0) + 2\Omega k_x \right], \quad (3.31)$$

$$A_{23} = -\frac{1}{k_x u_0 - \omega} \frac{k_x^2 + k_y^2}{\omega} \frac{\rho_0}{\rho_c}, \quad (3.32)$$

$$A_{34} = 1, \quad (3.33)$$

$$A_{41} = \frac{4\pi G \rho_c}{c^2}, \quad (3.34)$$

$$A_{43} = k_x^2 + k_y^2, \quad (3.35)$$

and others are zero, i.e., $A_{24} = A_{31} = A_{32} = A_{33} = A_{42} = A_{44} = 0$.

We integrate equation (3.25) and seek solutions satisfying the boundary conditions described in subsection 3.3. Then we obtain the dispersion relation, i.e., ω as a function of k_x and k_y for a given slab model.

3.3. Boundary Conditions

Since the gaseous slab is sandwiched and pressed together with two plane-parallel shock fronts, the boundary surface of our integration is assumed to be rigid and fixed [e.g. Stone (1970), Voit (1988)]. That is, we take into account the fact that the plain shock front is stable against perturbation (e.g. Landau and Lifshitz 1987). When a wavy perturbation is imposed along the shock front, the velocity component of gas parallel to the disturbed surface does not change across the shock front, but, the component normal to it decreases considerably due to the compression of gas. The resultant motion of the gas is divergent from the convex region (seen from the downstream) and similarly convergent to the concave one, restoring the distorted front surface to its original plane surface.

We allow of the pressure discontinuity δp due to the perturbation. According to equation (3.23),

it is written as

$$y_1(L) = \delta p. \quad (3.36)$$

The perturbed velocity normal to the surface, u_{z1} or y_2 , must be zero,

$$y_2(L) = 0. \quad (3.37)$$

The perturbed potential ψ_1 is continuous and smooth at $z = \pm L$. Solving the Poisson's equation in the space outside the slab, $\Delta \hat{\psi}_1 = 0$, we have

$$\psi_1(z) = \psi_1(L) \exp[-\sqrt{k_x^2 + k_y^2}(z - L)], \quad (3.38)$$

in $z > L$, and thus

$$\frac{d}{dz}\psi_1(L) = -\sqrt{k_x^2 + k_y^2}\psi_1(L). \quad (3.39)$$

Using equations (3.23) and (3.24), we have the boundary conditions on y_3 and y_4 ,

$$y_3(L) = \rho_c \psi_1(L), \quad (3.40)$$

and

$$y_4(L) = -\rho_c \sqrt{k_x^2 + k_y^2} \psi_1(L). \quad (3.41)$$

Equations (3.36), (3.37), (3.40), and (3.41) form a complete set of the boundary conditions at $z = L$. Note, however, that it includes two parameters, δp and $\psi_1(L)$, whose numerical choices are still free.

Similar boundary conditions are imposed at $z = -L$. In total the boundary conditions have four degrees of freedom as well as the differential equation (3.25) has.

3.4. Numerical Methods

Our method to obtain the numerical dispersion relation is essentially the same as those of Nakamura et al. (1991) and Matsumoto et al. (1994). We seek ω for which a solution of equation (3.25) satisfies the boundary conditions at $z = \pm L$ using a bisection method.

The general solution satisfying the boundary condition at $z = L$ is expressed as a linear combination of two linearly independent solutions,

$$\mathbf{y}(z) = \delta p \mathbf{y}^{(1)}(z) + \psi_1(L) \mathbf{y}^{(2)}(z) \quad (3.42)$$

where $\mathbf{y}^{(1)}(z)$ and $\mathbf{y}^{(2)}(z)$ are solutions of equation (3.25) and their values at $z = L$ are given by

$$\mathbf{y}^{(1)}(L) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad (3.43)$$

and

$$\mathbf{y}^{(2)}(L) = \begin{pmatrix} 0 \\ 0 \\ \frac{\rho_c}{-\rho_c \sqrt{k_x^2 + k_y^2}} \end{pmatrix}, \quad (3.44)$$

respectively. By the Runge-Kutta method we integrated equation (3.25) to obtain $\mathbf{y}^{(1)}(z)$ and $\mathbf{y}^{(2)}(z)$. The parameters, δp and $\psi(L)$ are left unfixed at this moment.

Similarly we obtain the general solution, $\mathbf{y}(z) = \delta p' \mathbf{y}^{(3)}(z) + \psi(-L) \mathbf{y}^{(4)}(z)$, which satisfies the boundary condition at $z = -L$. The two general solutions coincide each other at $z = 0$, and thus

$$\begin{pmatrix} c_{11} & \cdots & c_{14} \\ \vdots & \ddots & \vdots \\ c_{41} & \cdots & c_{44} \end{pmatrix} \begin{pmatrix} \delta p \\ \psi_1(L) \\ -\delta p' \\ -\psi_1(-L) \end{pmatrix} = \mathbf{0}, \quad (3.45)$$

with

$$c_{ij} \equiv y_i^{(j)}(0) \quad i, j = 1, 2, 3, 4. \quad (3.46)$$

Equation (3.45) has a non-trivial solution when and only when the determinant of the matrix $(c_{i,j})$ vanishes,

$$\det \begin{vmatrix} c_{11} & \cdots & c_{14} \\ \vdots & \ddots & \vdots \\ c_{41} & \cdots & c_{44} \end{vmatrix} = 0 \quad (3.47)$$

We computed the determinant as a function of ω and sought ω for which the determinant has a sufficiently small value. See Matsumoto et al. (1994) for the algorithm to obtain ω for the determinant has a smaller value successively.

4. Gravitational Instability of the Plane-Parallel Gaseous Slab

The dispersion relations between ω and $\mathbf{k} [\equiv (k_x, k_y)]$ are obtained for four cases with/without the shear and the rotation. First we study the gravitational instability of non-rotating slab without velocity shear in subsection 4.1. The effects of velocity shear and rotation are shown separately in subsections 4.2 and 4.3, respectively. We discuss the gravitational instability of a rotating slab with velocity shear in subsection 4.4.

We represent all the dispersion relation in a nondimensional form for extensive applications of the results. For this purpose we normalize velocity, time, and density, with the isothermal sound speed c , the free-fall time $1/\sqrt{2\pi G \rho_c}$, and the density ρ_c at $z = 0$, respectively. Thereby the length is automatically normalized with the density scaleheight, $H = c/\sqrt{2\pi G \rho_c}$. After the normalization, parameters are reduced to \mathbf{k}, L, U , and Ω .

4.1. Non-rotating Slab without Velocity Shear ($\Omega = U = 0$)

Figure 2 shows the squared growth rates $(-\omega^2)$ of the unstable perturbation as functions of k_x for $k_y = 0$. The thin and thick curves denote the dispersion relations for $L = 0.5$ and 1.0 , respectively. The growth rate vanishes at $k_x = 0$ and the perturbation is stable when the wavenumber exceeds the critical one, $k_x > k_{cr}$. The critical wavenumber is $k_{cr} = 0.74$ and 0.95 for $L = 0.5$ and 1.0 , respectively. The growth rate has its maximum $-\omega_{\max}^2 = 0.17$ and 0.34 at $k_{x \max} = 0.37$ and 0.46 , respectively for $L = 0.5$ and 1.0 .

Figure 2 suggests that $-\omega_{\max}^2$ and $k_{x \max}$ de-

Fig. 2.— Dispersion relations of the gravitational instability when $U = \Omega = 0$. Thick and thin lines are results for the fixed and free boundaries, respectively. We write adopted L in the figure.

Fig. 3.— The dependence of the maximum growth rates and their wavenumbers on the thickness of the slab for $U = \Omega = k_y = 0$. The meanings of the thick and thin lines are the same as in figure 2. Upper two lines represents squared maximum growth rates, while lower ones are the wavenumbers at the maximum growth rates.

crease as the slab thickness $2L$ decreases. Figure 3 confirms it. For a given k_x the mass contained in a wavelength decreases and hence the growth of the gravitational instability weakens as the slab becomes thinner. When the slab is thin, $0 \leq L \leq 0.5$, the dispersion relation shown in figure 2 and 3 can be reproduced approximately by the gravitational instability of the infinitely thin sheet of gas whose column density is given by equation (3.15) (e.g. Stone 1970, and Goldreich and Tremaine 1979):

$$\omega^2 = c^2 k_x^2 - 2\pi G \sigma k_x, \quad (4.1)$$

with

$$-\omega_{\max}^2 = \frac{(\pi G \sigma)^2}{c^2}, \quad (4.2)$$

and

$$k_{x \max} = \frac{\pi G \sigma}{c^2} \quad (4.3)$$

[We suspend the normalized expression for quantities in equations (4.1) through (4.3).]. Also for a larger L equations (4.2) and (4.3) reproduce the qualitative features of the dispersion relation but less accurately. At the limit of $L \rightarrow \infty$, $-\omega_{\max}^2$ and $k_{x \max}$ tend to 0.45 and 0.47, respectively (See also Simon 1965), while equations (4.2) and (4.3) give $-\omega_{\max}^2 = 1.0$ and $k_{x \max} = 1.0$.

The top diagram of figure 4 represents a cross sectional view of the fastest growing density perturbation with the growth rate, $-\omega_{\max}^2 = 0.17$, for the slab of $L = 0.5$. It delineates the one wave-length region $-\pi/k_{x \max} \leq x \leq \pi/k_{x \max}$. In order to see the vertical structure, we magnify the slab thickness about ten times in the bottom diagram. Contours denote the gas density. Arrows in the bottom panel shows the velocity distribution. We find that the perturbed velocities are predominantly parallel to the x -axis, and negligibly small in the z -axis, that is, the perturbation grows due mainly to the gas flow parallel to the slab. This tendency is more prominent when the slab is thinner.

4.2. Non-Rotating Slab with Velocity Shear ($\Omega = 0, U \neq 0$)

In the shearing slab both the gravitational and Kelvin-Helmholtz modes appear. The dispersion relations described here are about the gravitational mode, which continuously approach those in the previous subsection 4.1, as the shear becomes weaker. (We will discuss briefly the Kelvin-Helmholtz modes in subsection 6.2.)

Figure 5 shows the squared growth rate $-\omega^2$ as functions of k_x , for three cases that $M_{\parallel} \equiv u_0(L)/c = 0, 0.5$, and 1.0 , when $L = 1.0$ and $k_y = 0$. Both $-\omega_{\max}^2$ and $k_{x\max}$ decrease as M_{\parallel} increases. The dispersion relation remains parabolic-like on the $\omega^2 - k$ diagram. Figure 6 shows the detailed behaviour of $-\omega_{\max}^2$ against M_{\parallel} for the gaseous slabs of $L = 1.0, 0.5$, and 0.3 , when $\Omega = 0$ and $k_y = 0$. For a given L , $-\omega_{\max}^2$ is smaller when the shear is stronger. Particularly, when $M_{\parallel} \gg 1$, we have $-\omega_{\max}^2 \propto M_{\parallel}^{-2}$ (Compare with the thin line.).

According to our extensive calculation the numerical dispersion relation can be well approximated by

$$\omega^2 = c^2(1 + M_{\parallel}^2)k_x^2 - 2\pi G\sigma k_x. \quad (4.4)$$

Equation (4.4) implies that the velocity shear may be dealt with as an ensemble of vortices and the gaseous slab has the effective sound speed, $c\sqrt{1 + M_{\parallel}^2}$ [See equation (4.1).]. Note that the square of the effective sound speed is the arithmetic sum of the square of the sound speed and that of velocity shear. From equation (4.4) we obtain

$$-\omega_{\max}^2 = \frac{(\pi G\sigma)^2}{c^2(1 + M_{\parallel}^2)}, \quad (4.5)$$

and

$$k_{x\max} = \frac{\pi G\sigma}{c^2(1 + M_{\parallel}^2)}. \quad (4.6)$$

Equation (4.5) and (4.6) are also consistent with our numerical results, e.g. $-\omega_{\max}^2 \propto M_{\parallel}^{-2}$ when $M_{\parallel} \gg 1$.

Fig. 4.— The cross section of the slab perturbed by the fastest growing perturbation, when $U = \Omega = k_y = 0$.

Fig. 5.— Dispersion relations when $U \neq 0$ and $\Omega = 0$. The thin line denotes the relation for the non-shearing slab.

Fig. 6.— The dependence of the squared maximum growth rate on the shear magnitudes. The thin line is $\omega_{\text{max}}^2 \propto M_{\parallel}^{-2}$.

Figure 7 is the same as figure 4 except that the shear is present. $k_x = 0.285, k_y = 0.0, L = 1.0$, and $M_{\parallel} = 1.0$ are applied. The concentration of gas is similar to that in figure 4, but the perturbed gas spirals-in toward the point of maximum density. Note, however, that the spirals are elongated along the z -axis.

When $\Omega = 0$, the wave number, (k_x, k_y) appears in the matrix, \mathbf{A} , and in the boundary conditions only in the form of $k = \sqrt{k_x^2 + k_y^2}$ and $k_x u_0(z)$. Thus, we obtain a relation,

$$\omega(k_x, k_y; u_0, \Omega = 0) = \omega(k_x' = \sqrt{k_x^2 + k_y^2}, k_y' = 0; k_x u_0/k_x', \Omega = 0). \quad (4.7)$$

Using this relation we can obtain the growth rate for a mode having non-vanishing k_y . Hence, the discussions about the dispersion relation (figure 5) and the correlation between the maximum growth rate and the Mach number of the streaming motion parallel to the slab (figure 6) remain unchanged after the transformation from k_x and M_{\parallel} to k and $M_{\parallel} k_x/k$. The effective sound speed can be expressed as $c_{\text{eff}}^2 = c^2 [1 + k_x^2 M_{\parallel}^2 / (k_x^2 + k_y^2)]$. When $k_x = 0$ and $k_y \neq 0$, the growth rate is independent of the shear strength.

4.3. Rotating Slab without Shear ($\Omega \neq 0, U = 0$)

Figure 8 shows the squared growth rates $-\omega^2$ as a function of k_x for $\Omega = 0, 0.5$, and 1.0 . The parameters L and k_y are kept constant, $L = 1.0$ and $k_y = 0$. The growth rate decreases monotonically, as Ω increases. The rotation tends to suppress the gravitational instability. However, the decrease in the growth rate is smaller than that for a rotating disk of which rotation axis is the normal to the disk. The latter growth rate is approximated by

$$\omega^2 = c^2 k_x^2 - 2\pi G \sigma k_x + 4\Omega^2, \quad (4.8)$$

Fig. 7.— The cross section of the shearing slab perturbed by the fastest growing perturbation.

(see, e.g. Goldreich and Tremaine 1979) and the growth rate decreases by $4\Omega^2$ at a given k_x . Accordingly, as Ω increases, $k_{x\text{ cr}}$ decreases while $k_{x\text{ max}}$ remains unchanged. On the other hand, $k_{x\text{ max}}$ decreases in our model as Ω increases. The critical wavenumber $k_{x\text{ cr}}$ is independent of Ω in our model. As proved in appendix, the velocity perturbation vanishes for the marginally stable mode of $k_x = k_{x\text{ cr}}$ in our model. Then the Coriolis force does not work in the limit of $i\omega \rightarrow +0$. As Ω increases, the growth rate decreases but never vanishes as far as the mode is unstable at $\Omega = 0$.

The correlation between maximum growth rate $-\omega_{\text{max}}^2$ and the angular velocity, Ω , of the slab is given in figure 9 for $L = 0.1, 0.3, 0.5$, and 1.0 , when $k_y = 0$ and $U = 0$. As Ω increases, $-\omega_{\text{max}}^2$ decreases but a little and the decrease is very small for $L \leq 0.1$.

These new results are intrinsic to the rotating slab sandwiched and pressed together with two plane-parallel rigid surfaces; when the gaseous component is perturbed, it moves easily along the slab, but the Coriolis force due to this motion drives the gas to move normal to the slab. The narrowly-separated rigid surfaces, however, suppress this motion. The suppression is extremely strong, when the slab is thin, or the wavelength is long. Thus, the dispersion relations lack dramatic effect of the rotation in that case.

Figure 10a shows the perturbed density distribution and is the same as figure 4 except for $\Omega = 0.4$. We have taken a case of moderate slab thickness: $L = 1.0$, and assumed that $k_y = 0$. Then, the maximum growth rate is $-\omega_{\text{max}}^2 = 0.25$ and the corresponding wave number is $k_{x\text{ max}} = 0.43$. The global features are similar to those in figure 4, but we find that the iso-density contours are tilted due to the Coriolis force. The velocity normal to the slab is actually seen, derived by the Coriolis force. The concentration of gas is due primarily to the x -component of the perturbed velocity.

As the angular velocity increases the iso-density

Fig. 8.— Dispersion relations when $\Omega \neq 0$ and $U = 0$. The thin line corresponds to the non-rotating slab.

Fig. 9.— The dependence of the squared maximum growth rate on the angular velocity.

contours become more elongated and tilted, and then form two separated concentrations of gas (figure 10b).

So far we have dealt with the case of $k_x \neq 0$ and $k_y = 0$. When $k_y \neq 0$ and $k_x = 0$, or the wave vector is parallel to the rotation axis, we find the growth rates hardly change due to the rotation, since the perturbed flow is nearly parallel to the rotation axis, and then $2\mathbf{\Omega} \times \mathbf{u}_1 \simeq 0$.

4.4. Rotating Slab with Shear ($\Omega \neq 0, U \neq 0$)

This case would be the most realistic gaseous slab occurring at the interface of the obliquely-colliding clouds with supersonic velocity. As seen from equation (3.13), the equilibrium state is valid only in the parameter range of $\Omega U H / c^2 < 1$. Therefore, we present the results within it.

Figure 11 shows the dispersion relation between k_x and $-\omega^2$ for $M_{\parallel} = 1.0$, $\Omega = 0.5$, $k_y = 0.0$, and $L = 1.0$. For comparison, the dispersion relations for $(M_{\parallel}, \Omega) = (0.0, 0.0)$, $(1.0, 0.0)$, and $(0.0, 0.5)$ are shown with thin curves for comparison. The wavenumber and the thickness of the slab are kept constant $k_y = 0$ and $L = 1.0$ for all the thin and thick curves. The gravitational instability is suppressed additively by shear and rotation, and the critical wave number $k_{x\text{cr}}$ decreases as Ω increases, which is derived to be constant when $U = 0$.

Figure 12 shows the squared maximum growth rates as functions of Ω for $k_y = 0$ and $L = 1.0$. Three curves represent $-\omega_{\text{max}}^2$ for $M_{\parallel} = 0, 0.5$, and 1.0 . In this figure we confirm again that the rotation and shear suppress the gravitational instability. Note that the maximum growth rate is larger for a larger M_{\parallel} in the region of $\Omega \gtrsim 1.1$. This is because the column density σ is larger for a larger M_{\parallel} for given L [See equation (3.15)]. A larger column density brings a larger maximum growth rate.

Figure 13 is similar to both of figures 7 and

Fig. 10.— The cross section of the rotating slab disturbed by the fastest growing perturbation. The velocities seen from the rotating frame are delineated. We assume that $\Omega = 0.4$ and $k_x = 0.43$ in figure (a), and $\Omega = 1.0$ and $k_x = 0.30$ in figure (b).

Fig. 11.— The dispersion relations when $U \neq 0$ and $\Omega \neq 0$. Adopted M_{\parallel} and Ω for each line are written in the figure.

10a, but for $M_{\parallel} = 1.0$, $\Omega = 0.5$, $L = 1.0$, $k_x = 0.2$, and $k_y = 0$. The contour curve of constant density slants to the x -direction as in figure 10 a and the gas stream spirals into the point of the maximum density as in figure 7.

The dispersion relation between k_x and $-\omega^2$ in figure 12 cannot be empirically represented in a simple form including the parameters, the slab thickness $2L$, shear U , and angular velocity Ω . However, when the slab is a thin sheet of gas and the Coriolis force more suppressed, the dispersion relation reduced to that of the non-rotating slab in equation (4.4).

5. Jeans Mass with Rotation and Shear

When the gravitational instability grows and non-linear contraction proceeds, a high-density gaseous clump forms in the slab. When the slab rotates and has velocity shear, the Jeans wave length depends on the direction of the wave vector: The one parallel to the x -axis is given by $\lambda_{xJ} = 2\pi/k_{x\max}$ and equation (4.6), when $L \ll H_R$, and the other one, λ_{yJ} , parallel to the y -axis remain unaffected by the shear and rotation, and obtained from equation (4.3). Therefore, the most probable Jeans mass M_J would be

$$M_J = \frac{\lambda_{xJ}}{2} \frac{\lambda_{yJ}}{2} \sigma, \quad (5.1)$$

where $\lambda_{yJ} \equiv 2\pi/k_{y\max}$. Note that $\lambda_{x\max}$, $\lambda_{y\max}$, and σ are functions of the slab thickness $2L$, which increases with time. Using a similar equation, Whitworth et al. (1994) calculated the mass fragmenting in the shocked layer produced by the head-on collision.

Using t_m in equation (2.2), we define “the growth rate of the slab mass” as $\omega_m \equiv 1/t_m$. Assuming $d\sigma/dt \simeq \sigma/t$, where t denotes the time interval measured from the onset of the collision, we have

$$\omega_m = \frac{1}{t}, \quad (5.2)$$

which monotonically decreases with time. We note, while, that the maximum growth rate,

Fig. 12.— The dependence of the squared maximum growth rate on the angular velocity.

$-i\omega_{\max}$, of the instability increases with time, since σ and L increase as the collision proceeds [See also e.g. figure 3.]. At the beginning of the collision, $\omega_m \gg -i\omega_{\max}$, and the slab mass grows more rapidly than the perturbation does, and thus the slab is practically stable. When $-i\omega_{\max}$ exceeds ω_m , the perturbation grows appreciably. Equating ω_m with $-i\omega_{\max}$, we meet the time t_0 when the perturbation starts to grow,

$$t_0 = 1/(-i\omega_{\max}). \quad (5.3)$$

We have then the Jeans mass M_J (5.1) in which λ_{xJ} , λ_{yJ} , and σ are evaluated at $t = t_0$.

Using quantities defined in section 2, we have

$$\sigma = \frac{2\rho_c c t}{M_{\perp}}, \quad (5.4)$$

where we have used the relationships: $\sigma \simeq t d\sigma/dt$, $d\sigma/dt \simeq 2\rho_0 c M_{\perp}$, $\rho_1/\rho_0 \simeq M_{\perp}^2$, and $\rho_1 \simeq \rho_c$, when $L \ll H_R$. Substituting σ in equation (5.4) into (4.5), we have

$$-i\omega_{\max} = \frac{2\pi G \rho_c}{M_{\perp} \sqrt{1 + M_{\parallel}^2}} t. \quad (5.5)$$

From equation (5.3) and (5.5), t_0 is written as

$$t_0 = \left(\frac{M_{\perp}}{2\pi G \rho_c} \right)^{1/2} (1 + M_{\parallel}^2)^{1/4}. \quad (5.6)$$

Note that this is comparable to the free fall timescale, when $M_{\perp} \simeq 1$. Using the relations: $k_{x\max} = \pi G \sigma / c^2 (1 + M_{\parallel}^2)$ [from equation (4.6)], and $k_{y\max} = \pi G \sigma / c^2$ [from equation (4.3)], we have, respectively,

$$\begin{aligned} \lambda_{x\max} &= \frac{c M_{\perp}}{G \rho_c t_0} (1 + M_{\parallel}^2) = \\ &= 2\pi H M_{\perp}^{1/2} (1 + M_{\parallel}^2)^{3/4}, \end{aligned} \quad (5.7)$$

and

$$\begin{aligned} \lambda_{y\max} &= \frac{c M_{\perp}}{G \rho_c t_0} = \\ &= 2\pi H M_{\perp}^{1/2} (1 + M_{\parallel}^2)^{-1/4}, \end{aligned} \quad (5.8)$$

where we have used equation (3.14). These are roughly comparable to or greater than H , when $M_{\perp} \simeq 1$. Finally, we obtain the Jeans mass from equation (5.1),

$$M_J = 2\pi^2 \rho_c H^3 M_{\perp}^{1/2} (1 + M_{\parallel}^2)^{3/4}, \quad (5.9)$$

where we have used

$$\sigma(t = t_0) = 2\rho_c H M_{\perp}^{-1/2} (1 + M_{\parallel}^2)^{1/4}. \quad (5.10)$$

Note that when $M_{\perp} \gg 1$, we have $L(t = t_0) \ll H \simeq H_R$ from equation (5.10) and $\sigma \simeq 2\rho_c L$. When M_{\parallel} tends to zero, equations (5.6) through (5.9) are reduced to equation (1) in Whitworth et al.(1994), except for numerical factors.

The Jeans mass increases as U and Ω increase, and therefore the more massive gaseous clump would be generated in the gaseous slab at the interface of the obliquely colliding gas clouds. When the collision is strong and the resultant gaseous slab is thin, the perturbed motion normal to the shock front due to the Coriolis force is suppressed and thus the Jeans mass tends to that of the case of $U \neq 0$ and $\Omega = 0$ in subsection 4.2. We have derived it as analytical form in equation (5.9). In this case, as is shown in equations (5.7) and (5.8), the clump formed by the gravitational instability is elongated along the x -axis and rotates around the y -axis.

6. Conclusion and Discussion

We have worked on the gravitational instability of a gaseous slab with shear and rotation whose axis lies within and parallel to it. It is shown that more massive gaseous clumps are generated as the shear becomes stronger. The shear may be understood as ensemble of rotating eddies, contributing to increasing the effective sound speed. When the gaseous slab is thin and pressed together with two plane-parallel solid surfaces, the rotation affects little on the instability, because the perturbing force or the Coriolis force due to the rotation is directed only normal to them. We believe that the applied

plane-parallel solid surfaces are realistic enough to mimic the plane shock front which is stable against its deformation. We briefly touch upon below, what difference comes out from our results when we take the free boundary condition where the pressure always balance across the flexible and free surfaces.

6.1. On the Boundary Condition Mimicking the Shock Front

In figures 2 and 3 we reproduce Elmegreen and Elmegreen’s dispersion relation (1978) in thin lines, where $U = \Omega = 0$ and the free boundary condition are adopted. The dispersion relations in figure 2 show that the difference between the boundary conditions is more considerable as the slab thickness L decreases: compare (a,c) for $L = 1.0$ and (b,d) for $L = 0.5$. We find in figure 3 that the maximum growth rate $-\omega_{\max}^2$ and their corresponding wavenumber are independent of the boundary conditions as far as $L \gtrsim 1$. They, however, depend strongly on the boundary conditions when $L \lesssim 1$. When the boundary is “free”, $-\omega_{\max}^2$ does not change in the wider range of L , while $k_{x\max}$ increases as L decreases (Elmegreen and Elmegreen 1978; Lubow and Pringle 1993). As many authors suggested, these properties seems due to the instability whose density remain nearly constant while their configuration tend to round the structure (e.g. Vishniac 1983). Delineating the same figure as figure 4 but now for the free boundary, we confirm this instability in figure 14. The velocity field shows that the perturbed mass tend to round.

The fixed and free boundary conditions would be two extreme cases representing a surface, since the former mimics completely rigid one, and the latter flexible one. The real structure of the shock front must be in between these two extremes (See Welter 1982). We expect that the real boundary is closer to the rigid one when two clouds collide with high velocities and the shock wave is strong.

Fig. 13.— The cross section of the shearing and rotating slab disturbed by the fastest growing perturbation.

Fig. 14.— The same figure as figure 4 but for the free boundary. The wavenumber of this perturbation is now $k_x = 0.696$ (and $k_y = 0$).

6.2. Kelvin-Helmholtz Instability in the Slab with Shear and No Rotation

We have discussed exclusively the gravitational instability of the gaseous slab with shear and rotation. However, the basic equations in section 3 can automatically deal with the Kelvin-Helmholtz (K-H) instability, and it is appreciable when the wave length of the perturbation is shorter, $k_x \geq 1$, and the velocity gradient of the shear, du_0/dz , is larger. In order to study the Kelvin-Helmholtz instability we made a slab model with strong velocity shear in which the unperturbed shear flow is assumed to be

$$u_0(z) = U \tanh\left(\frac{z}{l}\right), \quad (6.1)$$

with $l \ll L$.

Figure 15 shows the relationship between k_x and $\text{Re}(-\omega^2)$, when $k_y = 0$, $L = 1.0$, $l = 1/30$, $M_{\parallel} = 2.0$, and $\Omega = 0$. We note that the ordinate is not measured in $-\omega^2$ as figure 2, 5, 8, and 11, but in $\text{Re}(-\omega^2)$, since $-\omega^2$ becomes the complex number. The solid lines denote the modes having real ω^2 while the dashed lines denote the mode having complex ω^2 . Thus each solid line is two-folded while each dashed line is four folded. In order to understand the network we labelled a solid line with a single character and a dashed line with two characters so that each symbol denotes a single continuous line. In the limit of $U \rightarrow 0$ the zigzag line $Ga - G - Gb - G - \cdot$ approaches the dispersion relation of the gravitational instability. The sequences, $Ga - a - ab - a - \cdot$, $\cdot - b - Gb - b - ab - \cdot$, and $\cdot - c - ac - c - bc - c - \cdot$ are the essentially the sound waves. These mode interact each other and form mixed modes, e.g., Ga , ab , bc , etc. This mode coupling is common in the dispersion relation of the K-H instability in the slab with boundary surfaces (Glatzel 1987a, b).

Figure 16 shows the density perturbations for the mode b of $[k_x, \text{Re}(-\omega^2)] = (3.2, 0.62)$, the mode c of $[k_x, \text{Re}(-\omega^2)] = (5.0, 0.47)$, and the mode bc of $[k_x, \text{Re}(-\omega^2)] = (4.1, -1.0)$. We find

Fig. 15.— Dispersion relations between k_x and $\text{Re}(-\omega^2)$. In the segments delineated by thick solid lines, $\text{Im} \omega = 0$, while by thick dashed lines, $\text{Im} \omega \neq 0$. We calculate dispersion relations in a right region of the thin line.

Fig. 16.— The cross section of the shearing slab disturbed by the growing sound waves. Iso-density contours are delineated. k_y, L, l, M_{\parallel} , and Ω are the same as in figure 15.

that these have periodic structure along the z -direction with “wavelength” $2L/n$: $n = 2$ for the mode b and $n = 3$ for the mode c . The mixed mode bc have characters of both the parent modes, and is similar to the mode b in the region $z > 0$ and while it is similar to the mode c in the region $z < 0$.

Since the mixed mode has a complex frequency, they grow or decay while oscillating in time, i.e., propagating with the phase speed $\text{Re}\omega/k_x$ in the x -direction. The density perturbation of the mixed mode has a smaller relative amplitude than the velocity perturbation of the mixed mode, $|\rho_1/\rho_0| < |\mathbf{u}_1/c|$. Thus, the growth of the mixed mode leads mainly to the diffusion of the velocity shear and does not likely to the formation of gravitationally bound systems.

6.3. Non-Linear Evolution of the Unstable Mass

We have shown that the gravitationally unstable clump collapses while rotating, when the slab has the shear or rotation. As many authors have pointed out, such a clump fragments into two parts during the collapse, and forms a binary system (or multiple system). Fujimoto and Kumai (1994) applies the results described in section 4 for the formation of binary star-clusters observed in the Magellanic Clouds.

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Appendix. Proof of $\mathbf{u}_1 = 0$ at Critical Wave Number when $M_{\parallel} = 0$

The present appendix is to show that $\mathbf{u}_1 = 0$ at the critical wave number \mathbf{k}_{cr} , i.e., when the perturbation is marginally stable ($\omega = 0$). We write the perturbed equations (3.16) through (3.20) in the non-dimensional form as in the text, and substitute equations (3.21) and (3.22) into them where we take $\omega = 0$ and $u_0 = 0$. We have then, writing \mathbf{k}_{cr} as \mathbf{k} for simplicity,

$$2\Omega u_{z1} = ik_x \left(-\psi_1 - \frac{\rho_1}{\rho_0} \right), \quad (\text{A.1})$$

$$0 = ik_y \left(-\psi_1 - \frac{\rho_1}{\rho_0} \right), \quad (\text{A.2})$$

$$-2\Omega u_{x1} = \frac{d}{dz} \left(-\psi_1 - \frac{\rho_1}{\rho_0} \right), \quad (\text{A.3})$$

$$ik_x \rho_0 u_{x1} + ik_y \rho_0 u_{y1} + \frac{d}{dz} (\rho_0 u_{z1}) = 0, \quad (\text{A.4})$$

and

$$\left[\frac{d^2}{dz^2} - (k_x^2 + k_y^2) \right] \psi_1 = 2\rho_1. \quad (\text{A.5})$$

(1) When $k_y \neq 0$, we have from (A.2),

$$\rho_1 = -\rho_0 \psi_1. \quad (\text{A.6})$$

Substituting equation (A.6) into (A.1) and (A.3), we have $u_{x1} = u_{z1} = 0$ since $\Omega \neq 0$ is presumed. Using $u_{x1} = u_{z1} = 0$, we have $u_{y1} = 0$, because of $k_y \neq 0$ in equation (A.4), and hence,

$$\mathbf{u}_1 = 0. \quad (\text{A.7})$$

(2) When $k_y = 0$ and $k_x \neq 0$, we combine equation (A.1) and (A.3) to remove the term $(-\psi_1 - \rho_1/\rho_0)$,

$$u_{x1} = \frac{d}{dz} \left(\frac{i u_{z1}}{k_x} \right), \quad (\text{A.8})$$

where we assume $\Omega \neq 0$ as before. Substituting equation (A.8) into (A.4), we have $u_{z1} = 0$. Then, equation (A.1) gives the relation (A.6) again. Substituting equation (A.6) into equations (A.1) and (A.3), we have

$$u_{x1} = u_{z1} = 0. \quad (\text{A.9})$$

Since we can choose $u_{y1} = 0$ without loss of generality, we have again equation (A.7).

REFERENCES

- Ashman, K. M., and Zepf, S. 1992, *ApJ* 384, 50.
- Elmegreen, B. G., and Elmegreen, D. M. 1978, *ApJ* 220, 1051.
- Fujimoto, M., and Kumai, Y. 1991a, *Ann. de Phys.* 16, 75.
- Fujimoto, M., and Kumai, Y. 1991b, in *The Magellanic Clouds*, IAU Symp. No. 148, ed. D. A. Hanes and D. Milne (D. Reidel Publishing Company, Dordrecht), p469.
- Fujimoto, M., and Kumai, Y. 1994, in preparation.
- Fujimoto, M., and Noguchi, M. 1990, *PASJ* 42, 505.
- Goldreich, P., and Tremaine, S. 1979, *ApJ* 233, 857.
- Gunn, J. E. 1980, in *Globular Clusters*, ed. D. Hanes and B. Hodge (Cambridge Univ. Press, Cambridge), p301.
- Glatzel, W. 1987a, *MNRAS* 225, 227.
- Glatzel, W. 1987b, *MNRAS* 228, 77.
- Hasegawa, T., Sato, F., Whiteoak, J. B., and Miyawaki, R. 1993, Submitted to *ApJL*.
- Kang, H., Shapiro, P. R., Fall, S. M., and Rees, M. J. 1990, *ApJ* 363, 488.
- Kumai, Y., Basu, B., and Fujimoto, M. 1993, *ApJ* 404, 144.
- Landau, L. D., and Lifshitz, E. M. 1987, *Fluid Mechanics* (2nd ed.; Pergamon Press, London), §90.
- Loren, R. B. 1976, *ApJ* 209, 466.
- Lubow, S. H., and Pringle, J. E. 1993, *MNRAS* 263, 701.
- Matsumoto, T., Nakamura, F., and Hanawa, T. 1994, *PASJ* 46, 243.
- Nakamura, F., Hanawa, T., and Nakano, T. 1991, *PASJ* 43, 685.
- Odenwald, S., Fischer, J., Lockman, F. J., and Stemwedel, S. 1992, *ApJ* 397, 174.
- Sabano, Y., and Tosa, M. 1985, in *Theoretical Aspects on Structure, Activity, and Evolution of Galaxies, II*, ed. S. Aoki and Y. Yoshii (Tokyo Astronomical Observatory, Tokyo), 45.
- Scoville, N. Z., Sanders, D. B., and Clemens, D. P. 1986, *ApJL* 310, L77.
- Serabyn, E., Güsten, R., and Schulz, A. 1993, *ApJ* 413, 571.
- Simon, R. 1965, *Ann. Astrophys.* 28, 40.
- Spitzer, L., Jr. 1942, *ApJ* 95, 329.
- Stone, M. E. 1970, *ApJ* 159, 277.
- Struck-Marcell, C. 1982, *ApJ* 259, 116.
- Vishniac, E. T. 1983, *ApJ* 274, 152.
- Voit, G. M. 1988, *ApJ* 331, 343.
- Welter, G. L. 1982, *A&A* 105, 237.
- Whitworth, A. P., Bhattal, A. S., Chapman, S. J., Disney, M. J., and Turner, J. A. 1994, *MNRAS* 268, 291.